Mode shape and natural frequency identification for seismic analysis from background vibration

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ABSTRACT: Background vibration data from several CANDU plants was examined and found to be suitable for determining modal parameters for seismic analysis of major plant equipment. A theoretical analysis based on spectral techniques in random vibration and curve fitting was developed to determine the mode shapes, natural frequencies and damping ratios from acceleration measurements without any knowledge of the excitation forces. The analytical method was verified in the laboratory on a randomly excited structure. The results show that this method of modal analysis from background vibration can be used to confirm seismic analysis of major equipment in a CANDU plant.

1 INTRODUCTION

The current practice for the design of structures and major equipment such as steam generators, calandria and fueling machines in a CANDU plant against damage due to earthquake includes numerical modeling to determine the dynamic characteristics. A field verification of the analytical models is desirable for a comprehensive seismic design. A common practice for such verification requires artificial excitation of the equipment by large exciters. This approach may require a plant shut down and is expensive, and runs the risk of causing structural damage during testing. The use of background vibration to determine the dynamic characteristics is therefore appealing.

Wind induced vibration has been used to predict the dynamic properties of tall structures (1,2,3,4). Luz, et al (5) used excitation by environmental noise for identification of natural frequencies and mode shapes in a nuclear power plant. McLamore, et al (6) used traffic and wind-induced vibration to determine the dynamic characteristics of two suspension bridges. In this work the response measurements were taken relative to an arbitrarily chosen reference measurement point. Spectral techniques were then used to estimate natural frequences and mode shapes. In this paper, the concept of structural relative displacement is further developed to allow a more accurate determination of dynamic characteristics.

Background vibration data from the various CANDU and other power plants of Ontario Hydro is studied to establish if this data would be useful for further analysis. Analysis using spectral techniques in random vibration are then developed to determine the structural response from which natural frequencies and mode shapes are determined manually. A curve-fitting technique is then used interactively to determine the natural frequencies, mode shapes and damping ratios more accurately(7).

The method of analysis was verified in the laboratory. The experimental results are presented and compared with other methods of analysis. The results show that the method for analysis of background vibration is accurate and can be practically implemented in the field.

2 BACKGROUND VIBRATION IN A CANDU PLANT

Background vibration of power plant equipment results from vibrations transmitted through structural supports, pipe and duct connections, and vibrations generated within the equipment. The main sources of such vibrations are the rotating equipment such as fans, compressors, pumps, turbo-generator, and flow through pipes and ducts. The excitation forces resulting in vibration of equipment can be considered to be a combination of harmonic and random character. It is the relative relationship between the natural frequencies of the structure and the frequency content of the excitation forces which determines how well the various vibration modes of the structure are excited. For example, a structural mode at 2 Hz will not be excited as well as a mode at 50 Hz if the major frequency content of the excitation is in the 60 to 100 Hz range. In seismic analysis the natural frequencies in the range of 2 to 50 Hz are of interest and therefore it is a prerequisite that the background vibration frequencies are in the vicinity of this range.

The other prerequisite for utilizing background vibration is that the amplitude of vibration should be significantly higher than the instrumentation noise.

To determine whether background vibration in a CANDU plant meets these two prerequisites, plant vibration data from Ontario Hydro was studied. This data had been recorded during routine troubleshooting and commissioning in the plant and was not specifically carried out for this study. A majority of the accelerations were recorded in the 2 to 20 kHz range and were of no interest for seismic analysis. However several data records show response frequencies in the 0 to 50 Hz range which are of interest for seismic analysis. These include vibrations measured at the reactivity and liquid zone control unit (0.38 to 0.58 g), feeder pipe (0.5 g) and guide tube (1.25 g). A review of this data indicated that a sufficiently high level of background vibration of interest for seismic analysis exists in a CANDU plant.

3 THEORY

The structure is considered as a lightly damped N-degree of freedom linear system represented by a mass matrix [M], damping matrix [C], and stiffness matrix [K]. The motion of the structure when excited by forces $\{F\}$ is governed by a a system of equations.

$$[M]\{\ddot{x}\} + [C]\{\dot{x}\} + [K]\{x\} = \{F\} \qquad (1)$$

Assuming that the excitation forces, F_i are unknown and the only available data are the structural accelerations $\ddot{x}_i$ measured at N locations on the structure. Then, by selecting a reference node o on the structure and introducing the relative displacements $x_i^1 = x_i - x_o$ into Equation (1) we obtain

$$[M^1]\{\ddot{x}^1\} + [C^1]\{\dot{x}^1\} + [K^1]\{x^1\} = \{F^1\} \qquad (2)$$

The new system matrices $[M^1]$, $[C^1]$ and $[K^1]$ are the same as the corresponding matrices [M], [C], and [K] except for the deletion of the one row and one column corresponding to the reference node o. The elements of the new force vector $\{F^1\}$ are

$$F_i^1 = F_i - m_i\ddot{x}_o - c_i\dot{x}_o - k_i x_o \qquad (3)$$

where m_i = mass associated with the node i

c_i = sum of the elements of the damping matrix [C] in the ith row

k_i = sum of the elements of the stiffness matrix [K] in the ith row.

The static component $k_i x_o$ and the viscous component $c_i\dot{x}_o$ of the force F_i^1 (Equation 3) are assumed to be small relative to the dynamic component $m_i\ddot{x}_o$ and can be neglected, thus

$$\{F_i^1\} = \{F_i\} - \{m_i\}\ddot{x}_o \qquad (4)$$

The system of coupled Equations (2) can be uncoupled to the form

$$[M_r']\{\ddot{y}_r\} + [C_r']\{\dot{y}_r\} + [K_r']\{y_r\} = \{F_r'\} \qquad (5)$$

by using the transformation

$$\{x'\} = [\phi_{ir}]\{y_r\} \qquad (6)$$

where $\{y_r\}$ is a new set of modal coordinates and $[\phi_{ir}]$ is the modal matrix where the r^{th} column represents the normalized mode shape vector for the r^{th} mode with natural frequency f_r.

$[M_r']$, $[C_r']$, $[K_r']$ are diagonal matrices where M_r', C_r', K_r' and F_r' are known as the generalized-mass, -damping, -stiffness and -force vector corresponding to the r^{th} mode, respectively.

Equation (5) represents equations of motion for single degree of freedom in the modal coordinate y_r (r=1,2,3.....R) and therefore by Fourier transformation to the frequency domain

$$Y_r(f) = H_r(f)\, F_r'(f)/M_r' \qquad (7)$$

where $H_r(f)$ is the frequency response function for the r^{th} mode and is defined by

$$H_r(f) = 1/[(2\pi)^2(f_r^2 - 2j\xi_r f_r f - f^2)] \qquad (8)$$

where $j = \sqrt{-1}$
ξ_r = damping ratio

From Equations (6) and (7) the structural relative displacement at node i in the frequency domain is given by

$$X_i(f) = \sum_{r=1}^{R} \phi_{ir} H_r(f) F_r'(f)/M_r' \qquad (9)$$

(the superscript 1 to represent relative displacement is deleted for convenience)

In the same way the structural relative acceleration in the frequency domain can be obtained

$$\ddot{X}_i(f) = \sum_{r=1}^{R} \phi_{ir}\, I_r(f)\, F_r'(f)/M_r' \qquad (10)$$

where $I_r(f) = -(2\pi f)^2 H_r(f)$ is the structural inertance in the r^{th} mode. The generalized exciting force $F_r'(f)$ in the r^{th} mode consists of two components

$$F_r' = \{\phi_{ir}\}^T \{F_i^1\}$$

$$= \sum_{k=1}^{N} \phi_{kr} F_k(f) - \ddot{X}_o(f) \sum_{k=1}^{N} \phi_{kr} m_k \qquad (11)$$

The first term represents the unknown exciting forces and the second one represents the fictitous component resulting from the consideration of relative displacements. From Equations (10) and (11) the structural response acceleration can thus be expressed as

$$\ddot{X}_i(f) = \qquad (12)$$

$$\sum_{r=1}^{R} \phi_{ir} I_r(f) \left[\sum_{k=1}^{N} \phi_{kr} F_k(f) - \ddot{X}_o(f) \sum_{k=1}^{N} \phi_{kr} m_k \right] / M_r'$$

If the unknown exciting force F_k as well as the reference acceleration $\ddot{x}_o$ have random characteristics, then the relation between structural response acceleration $\ddot{X}_i(f)$ and the reference acceleration $\ddot{X}_o(f)$ can be expressed in terms of spectral densities G calculated directly using Fast Fourier Transformation (FFT).

$$G_{oi}(f) = \frac{2\Delta t}{N'} \left[\bar{\ddot{X}}_o(f)\, \ddot{X}_i(f) \right]$$

$$G_{oo}(f) = \frac{2\Delta t}{N'} \left[\bar{\ddot{X}}_o(f)\, \ddot{X}_o(f) \right] \qquad (13)$$

$$G_{ok}(f) = \frac{2\Delta t}{N'} \left[\bar{\ddot{X}}_o(f)\, F_k(f) \right]$$

where Δt and N' are the time interval and number of acceleration records, respectively.

Performing these operations on Equation (12) we obtain

$$G_{oi}(f) = \sum_{r=1}^{R} \phi_{ir} I_r(f) \left[\sum_{k=1}^{N} \phi_{kr} G_{ok}(f) - G_{oo}(f) \sum_{k=1}^{N} \phi_{kr} m_k \right] / M_r' \qquad (14)$$

If the reference node o is chosen such that the acceleration x_o is statistically uncorrelated with any of the unknown exciting forces F_k then the cross-spectral densities $G_{ok}(f)$ are zeros. Thus the following simplified relation can be obtained

$$G_{oi}(f) = \sum_{r=1}^{R} \phi_{ir} I_r(f)\, M_r^* G_{oo}(f) \qquad (15)$$

where $M_r^* = \left[\sum_{k=1}^{N} \phi_{kr} m_k \right] / M_r'$ is a constant associated with the mode r. Having spectral densities G_{oi} and G_{oo}, we compute the ratio

$$a_i(f) = \frac{G_{oi}(f)}{G_{oo}(f)} = -\sum_{r=1}^{R} A_{ir} I_r(f) \qquad (16)$$

where $A_{ir} = \phi_{ir} M_r^*$ is the amplitude of $a_i(f)$ at the node i on the structure corresponding to the r^{th} mode. The amplitudes

A_{ir}, natural frequencies f_r and damping ratios ξ_r are then determined using the curve-fitting technique.

The success of modal identification using this method depends considerably on the proper choice of the reference node o. When the choice is not obvious, the coherence function

$$\gamma_{oi}(f) = \frac{\left| G_{oi}(f) \right|^2}{G_{ii}(f)\, G_{oo}(f)} \qquad (17)$$

should be examined to ensure that it is a maximum.

4 CURVE FITTING

The objective of curve fitting is to determine the modal parameters: amplitude A_{ir}, natural frequences f_r and the damping ratios ξ_r from the digital representation of the ratio $a_i(f) = G_{oi}(f)/G_{oo}$ given by Equation (16). The ratio, $a_i(f)$ is obtained as a series of complex numbers $a_i(f_k) = Re[a_i(f_k)] + jIm[a_i(f_k)]$ for each node i, and f_k are the points on the frequency scale. Equation (16) relates the modal parameters to the ratio $a_i(f)$ obtained from measurements. By substituting from Equation (8) and (10) for inertance $I_r(f)$, Equation (16) is rewritten in the following form

$$a_i(f) = \sum_{r=1}^{R} \frac{A_{ir}}{1 - 2j\xi_r(f_r/f) + (f_r/f)^2} \qquad (18)$$

which can be used for identification of all the modal parameters at the node i on the structure.

A modification of the least squares method is used for fitting a curve to the ratio $a_i(f)$ calculated at each node i separately. The method minimizes the function

$$S^2 = \sum_k \left| a(f_k) - a_k \right|^2 \qquad (19)$$

with respect to the unknown modal parameters A_{ir}, f_r and ξ_r, where a_k is the numerical representation of a(f) calculated by Equation (16) and $a(f_k)$ is the value on the curve to be fitted and defined by Equation (18).

5 EXPERIMENTAL VERIFICATION

Figure 1 shows the model structure which was tested on a horizontal shake table in the laboratory. The dynamic characteristics of the model were determined analytically to ensure that several natural frequencies exist in the 2 to 50 Hz range and that a couple of natural frequencies are closely spaced.

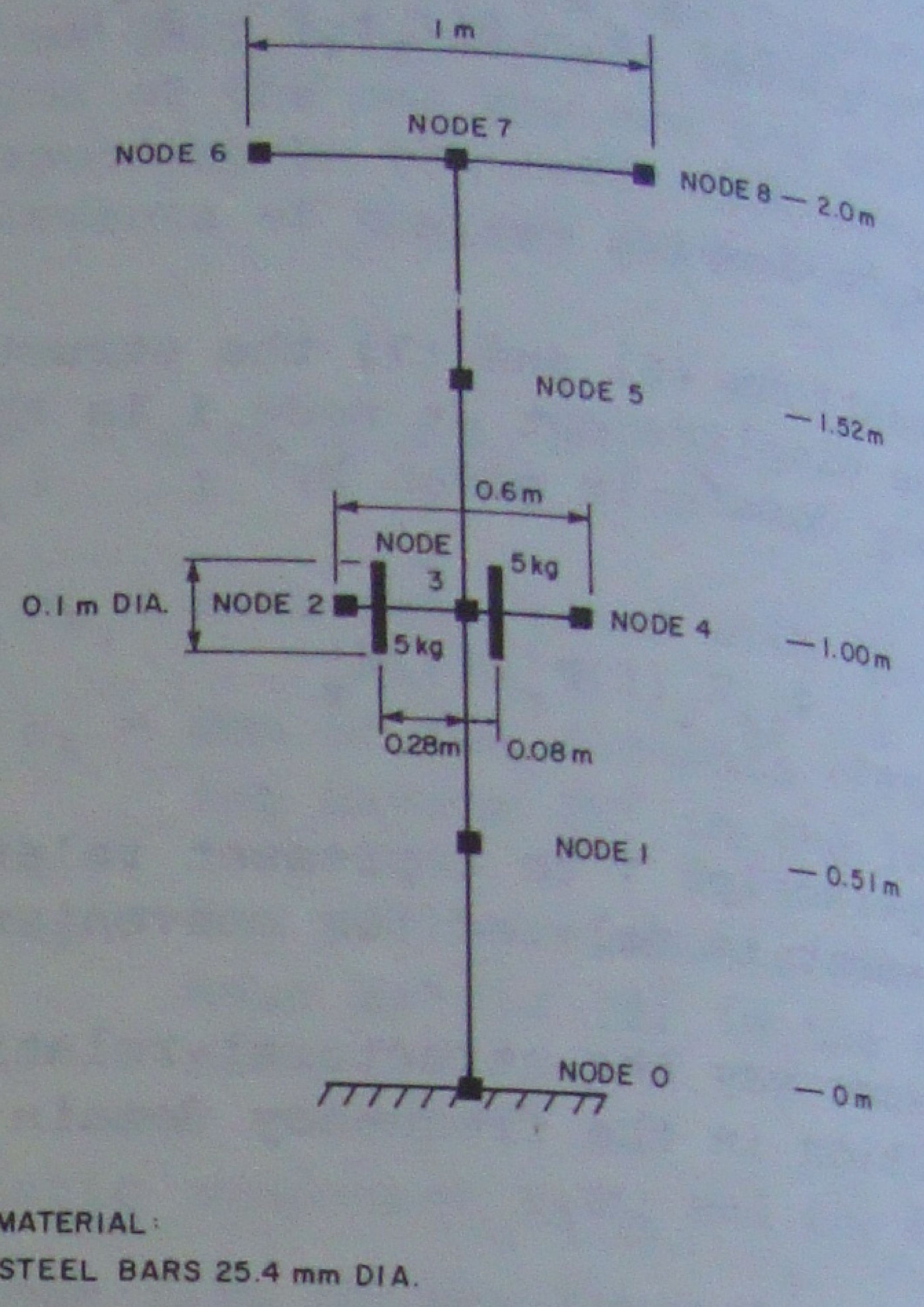

FIG. 1 BEAM WITH CROSS BARS AND TWO 5kg WEIGHTS

The general arrangement of the experiment is shown in Figure 2.

The structure was excited by random vibration of the shake table in the 2 to 50 Hz range with a peak displacement of 2.5 mm. The horizontal accelerations perpendicular to the paper at four positions on the structure were measured at 0.01 s intervals for 150 s and stored. The position of the accelerometer located on the shake table was considered as the reference node. The tests were repeated several times without changing the position of the accelerometers. The position of three of the accelerometers was changed twice while keeping the reference accelerometer on the shake table and the tests repeated.

The stored acceleration data was then reduced to give acceleration at each node relative to the reference node on the shake table. The spectral density functions were

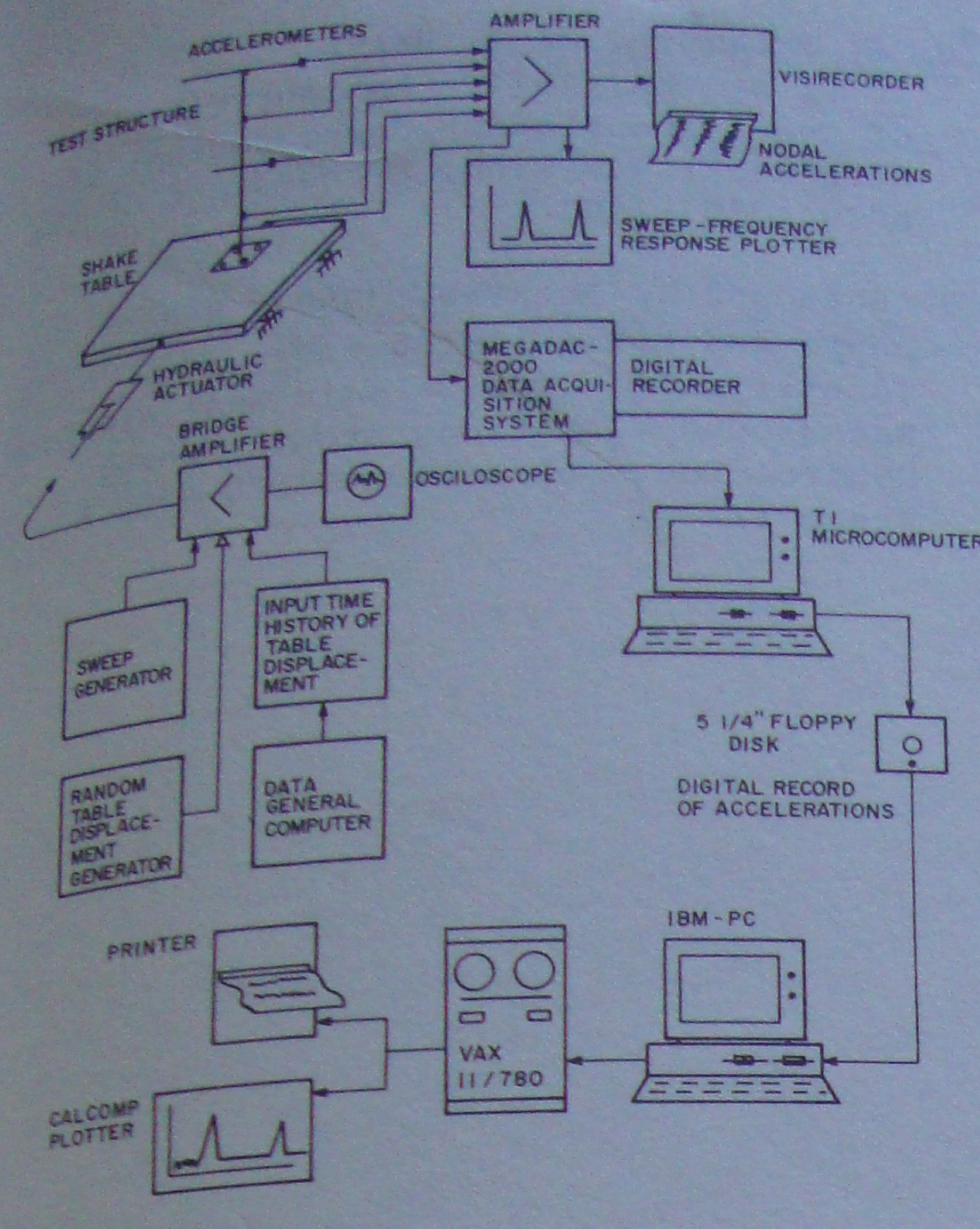

FIG. 2 GENERAL ARRANGEMENT OF THE EXPERIMENT AND DATA ANALYSIS

calculated by using a FFT algorithm. The acceleration records were partitioned into samples of 512 points long. Another 512 points with zero magnitude were added to each sample for better frequency resolution and the resultant frequency interval for spectral density estimates was 0.098 Hz. At each node the ratio of the cross spectral density function of the relative nodal acceleration with the reference acceleration and the spectral density of the reference acceleration was then calculated.

The interactive curve-fitting technique was used to fit a curve to the computed spectral density ratios at each node (Figures 3 and 4) from which the modal amplitude at that node, natural frequency and damping ratio were calculated.

The modal amplitudes for the various modes were then combined to constitute the mode shapes.

The natural frequencies and mode shapes were also extracted manually from the spectral density ratio plots and also calculated by a finite element program SAP4. A comparison of the natural frequencies, and mode shapes calculated by the three

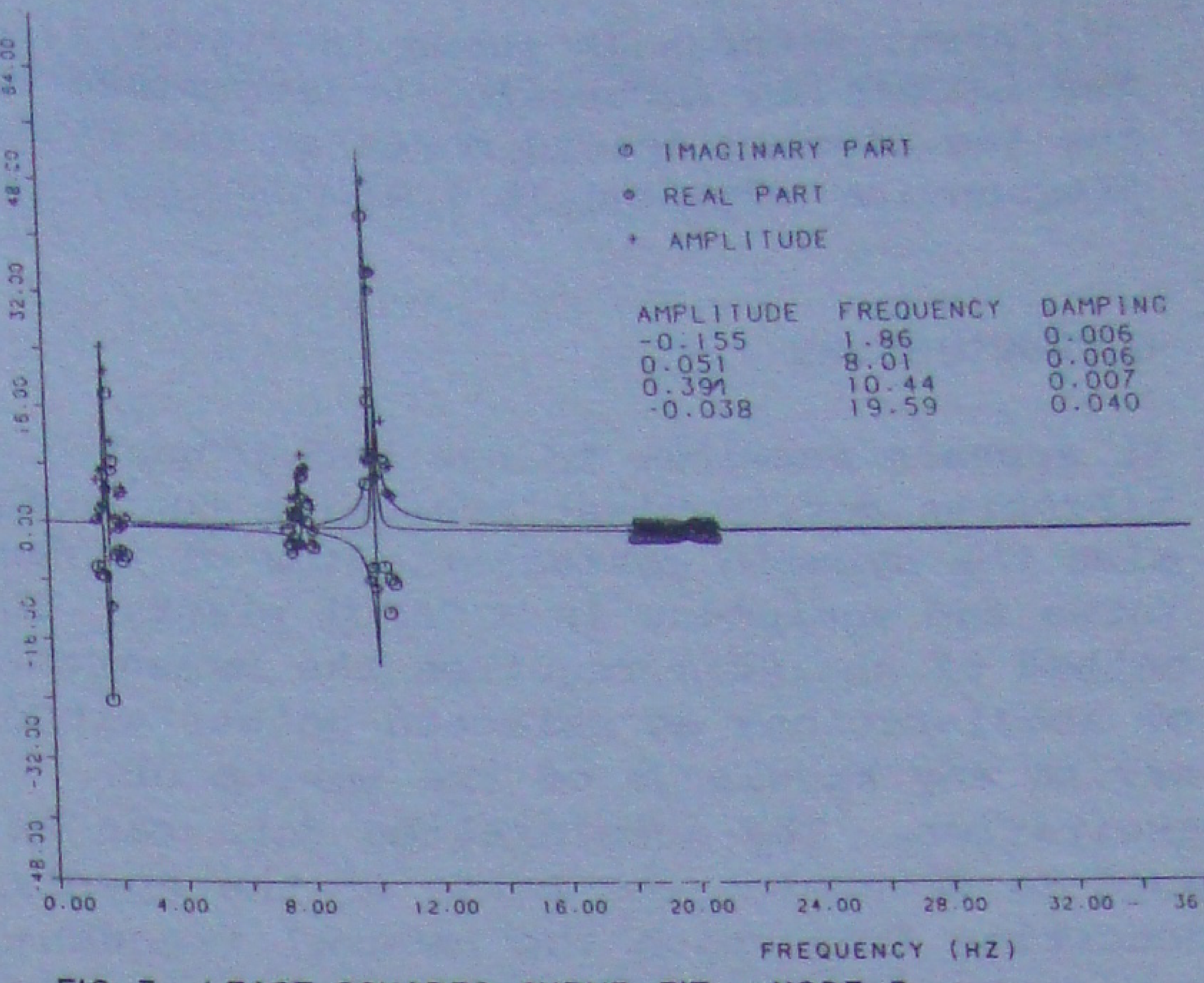

FIG. 3 LEAST SQUARES CURVE FIT - NODE 3

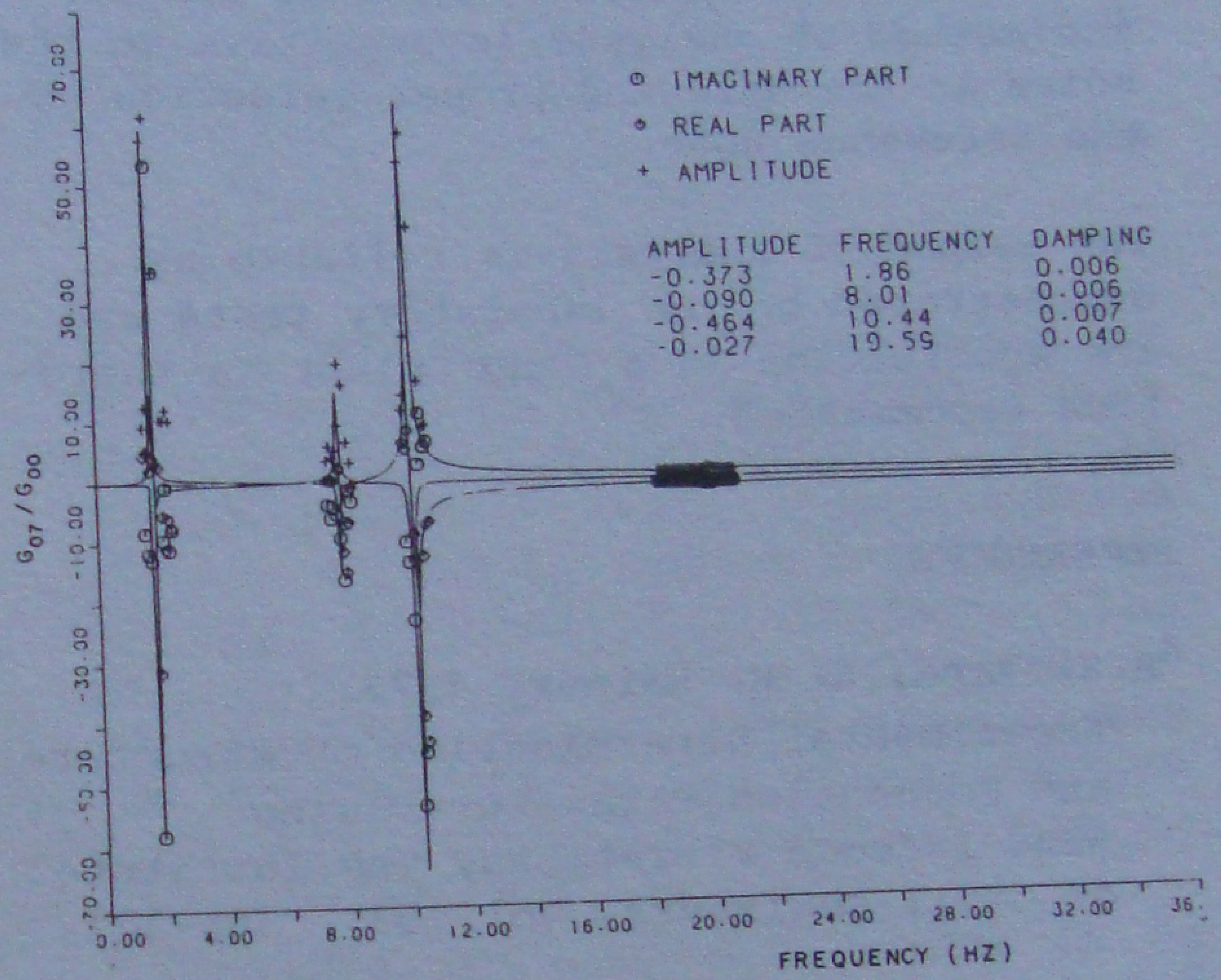

FIG. 4 LEAST SQUARES CURVE FIT - NODE 7

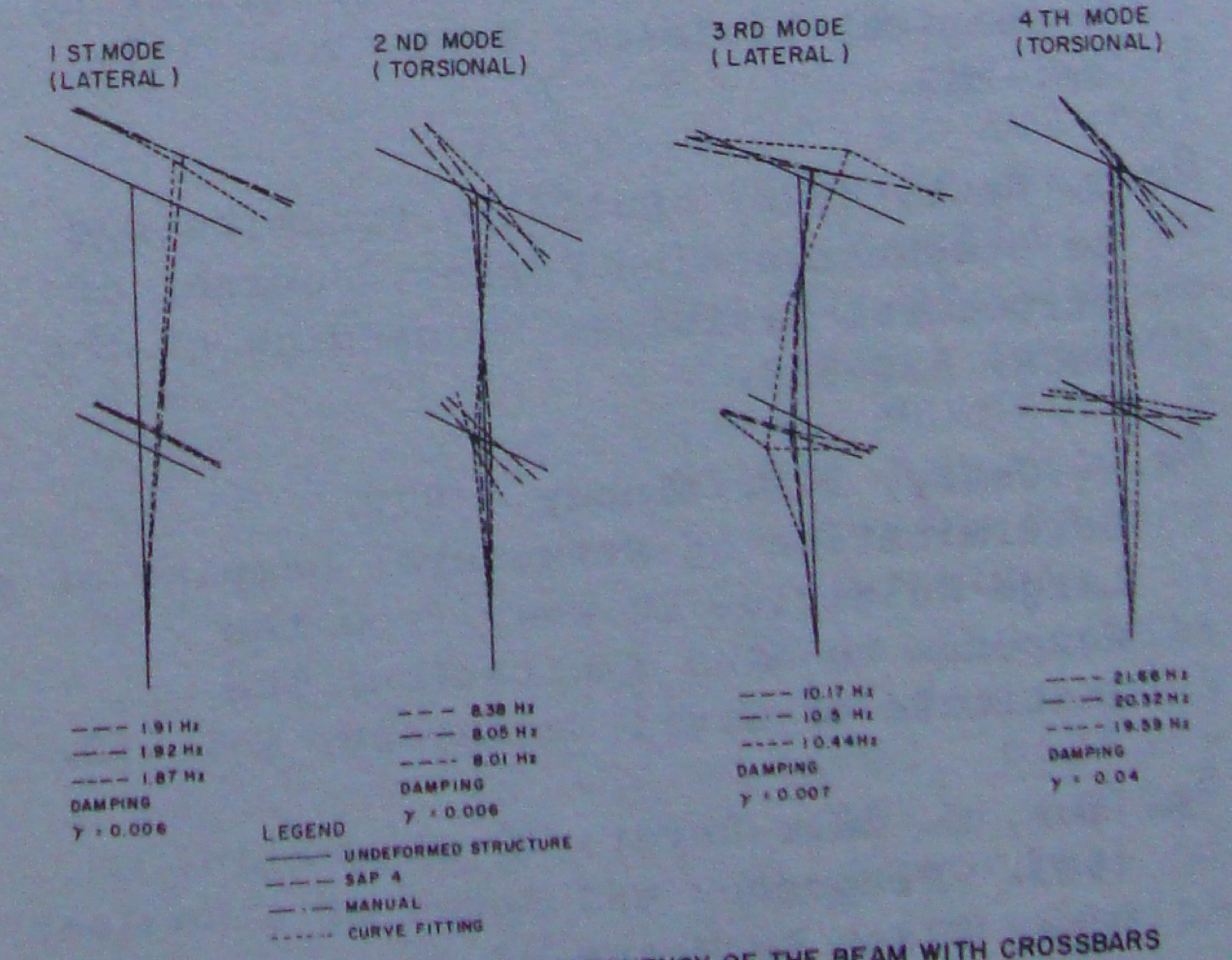

FIG. 5 MODE SHAPE AND NATURAL FREQUENCY OF THE BEAM WITH CROSSBARS

different methods is shown in Figure 5. The method has successfully determined even the two closely spaced modes at the natural frequencies of 8 and 10.4 Hz.

6 CONCLUSIONS

It appears feasible to use background vibration during plant operation to determine the dynamic characteristics of structures and equipment in a CANDU plant. The method of analysis requires the measurement of accelerations at selected points without having any knowledge of the source of excitation. The acceleration data can be used to determine the ratio of the spectral densities from which the natural frequencies and mode shapes can be extracted manually. A more elaborate curve-fitting technique can be used interactively to determine the damping ratios in addition to the natural frequencies and mode shapes. The method of analysis is sensitive to the noise in the data and proper selection of the reference node.

The method of analysis is reliable as demonstrated by the laboratory tests and its application in a CANDU plant is therefore recommended.

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